

Kantowski-Sachs Minimally Interacting Holographic Dark Energy Cosmological Model in Saez-Ballester Theory of Gravitation

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Abstract— A spatially homogeneous and anisotropic Kantowski-Sachs cosmological model is investigated in scalar tensor theory of gravitation proposed by Saez-Ballester, when universe is filled with minimally interacting fields; matter and holographic dark energy components. The solution of the field equations is obtained using the physical condition that the shear scalar is proportional to the expansion scalar. The physical behavior of the model is also discussed.

Keywords: Kantowski-Sachs universe, Holographic dark energy, Saez- Ballester theory of gravitation.

1 INTRODUCTION

It has been believed that the universe is undergoing a phase of accelerated expansion, which is indicated by the observational data from the cosmic microwave background (CMB) [1] and observations of Type Ia supernovae (SNe) [2] and large scale structure (LSS) [3]. The most accepted interpretation in context of Einstein's general relativity is that this phase is driven by an unknown component called dark energy. The Wilkinson microwave anisotropy probe (WMAP) [4] satellite experiment indicates that the universe is spatially flat on a large scale and the dark energy, dark matter and baryon matter in the universe make up about 73%, 23% and 4% respectively.

Several modifications of general relativity provide natural gravitational alternative for dark energy to explain early deceleration and late time acceleration behavior of the universe. Among this modifications, the scalar tensor theories of gravity proposed by Brans and Dicke [5], Saez and Ballester [6] and the modified theories of gravity like f(R) gravity proposed by Carroll *et al.* [7], f(R,T) gravity proposed by Harko *et al.* [8] are noteworthy.

In Saez-Ballester theory, the metric is coupled with a dimensionless scalar field in a simple manner which gives a satisfactory description of the weak fields. In spite of the dimensionless character of the scalar field an antigravity regime appears in the theory. Also this theory suggests a possible way to solve the "missing matter problem" in non-flat FRW cosmologies.

A new alternative to the solution of dark energy problem may be found in the 'Holographic principle' [9]. Number of cosmologists [10-13] have investigated several aspects of holographic dark energy. Holographic dark energy in non-flat universe has been extensively studied by Setare [14-15]. Sarkar [16] has investigated holographic dark energy model in Bianchi type-I universe with linearly varying deceleration parameter and established a correspondence with generalized Chaplygin gas models of the universe. Adhav *et al.* [17] have

obtained interacting dark matter and holographic dark energy model in an anisotropic universe. Samantha [18] has obtained holographic dark energy cosmological model with quintessence in Bianchi type-V space time. Kiran *et al.* [19-20] have investigated Bianchi type-V minimally interacting holographic dark energy models in Brans Dicke and Saez Ballester theories of gravitation. Recently, Reddy *et al.* [21] have obtained Kaluza Klein minimally interacting holographic dark energy model in a scalar tensor theory of gravitation.

In this paper, Kantowski-Sachs space time has considered when universe is filled with minimally interacting fields: matter and holographic dark energy components in the scalar tensor theory of gravitation proposed by Saez-Ballester. This work is organized as follows: In Section 2, the model and field equations have been presented. The field equations have been solved by applying special form of deceleration parameter in Section 3. The physical and geometrical behavior of the model has been discussed in Section 4. In the last Section 5, concluding remarks have been expressed.

2. METRIC AND FIELD EQUATIONS:

We consider spatially homogeneous and anisotropic Kantowski-Sachs space time described by the metric

$$ds^2 = dt^2 - A^2 dr^2 - B^2 (d\theta^2 + \sin^2 \theta d\phi^2), \quad (1)$$

where A, B are functions of cosmic time t . Saez-Ballester field equations for the combined scalar and tensor fields are given by

$$R_{ij} - \frac{1}{2} g_{ij} R - \omega \phi^n (\phi_{,i} \phi_{,j} - \frac{1}{2} g_{ij} \phi_{,k} \phi^{,k}) = (T_{ij} + \bar{T}_{ij}), \quad (2)$$

where R_{ij} is the Ricci tensor, R is the Ricci scalar, ω and n are arbitrary dimensionless constants and $8\pi G = c = 1$ in the relativistic units.

The energy-momentum tensor for matter and the holographic dark energy are defined as

$$T_{ij} = \rho_m u_i u_j$$

$$\text{and } \overline{T}_{ij} = (\rho_\lambda + p_\lambda)u_i u_j + g_{ij} p_\lambda, \quad (3)$$

where ρ_m, ρ_λ are the energy densities of matter and the holographic dark energy and p_λ is the pressure of holographic dark energy.

Also, the scalar field ϕ satisfies the following equation

$$2\phi^n \phi_{;i}^i + n\phi^{n-1} \phi_{,k} \phi^{,k} = 0. \quad (4)$$

The energy conservation equation is

$$T_{;j}^{ij} + \overline{T}_{;j}^{ij} = 0. \quad (5)$$

In a co-moving coordinate system, the field equations (2), for the metric (1), using equation (3) be given by

$$\frac{\dot{B}^2}{B^2} + 2\frac{\dot{A}\dot{B}}{AB} + \frac{1}{B^2} + \frac{\omega}{2}\phi^n \dot{\phi}^2 = (\rho_m + \rho_\lambda), \quad (6)$$

$$\frac{\dot{B}^2}{B^2} + 2\frac{\dot{B}}{B} + \frac{1}{B^2} - \frac{\omega}{2}\phi^n \dot{\phi}^2 = -p_\lambda, \quad (7)$$

$$\frac{\dot{B}}{B} + \frac{\dot{A}}{A} + \frac{\dot{A}\dot{B}}{AB} - \frac{\omega}{2}\phi^n \dot{\phi}^2 = -p_\lambda, \quad (8)$$

$$\frac{\ddot{\phi}}{\phi} + \frac{\dot{\phi}}{\phi} \left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B} \right) + \frac{n}{2} \frac{\dot{\phi}^2}{\phi^2} = 0, \quad (9)$$

where an overhead dot denotes differentiation with respect to t .

Using barotropic equation of state $p_\lambda = \omega\rho_\lambda$, we can write the continuity equation (5) of the matter and dark energy as

$$\dot{\rho}_m + \left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B} \right) \rho_m + \dot{\rho}_\lambda + \left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B} \right) (1 + \omega)\rho_\lambda = 0. \quad (10)$$

3. SOLUTION OF FIELD EQUATIONS:

The field equations (6) to (8) are a system of four highly non-linear differential equations in seven unknowns $A, B, \rho_m, \rho_\lambda, p_\lambda, \omega$ and ϕ . The system is thus initially undetermined. We need three extra physical conditions to solve the field equations completely.

(i) We are considering the minimally interacting matter and holographic dark energy components. Hence both components conserve separately so that we have (Sarkar [22], Kiran *et al.* [20])

$$\dot{\rho}_m + \left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B} \right) \rho_m = 0, \quad (11)$$

$$\text{and } \dot{\rho}_\lambda + \left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B} \right) (1 + \omega)\rho_\lambda = 0, \quad (12)$$

$$\text{where } \omega = \frac{p_\lambda}{\rho_\lambda}. \quad (13)$$

(ii) The expansion scalar (θ) is proportional to the shear scalar (σ) [23], which leads to

$$A = B^m, (m \neq 1). \quad (14)$$

(iii) The law of variation of Hubble's parameter proposed by

Berman [24] is modified by Singha and Debnath [25] called as special form of deceleration parameter defined by

$$q = \frac{-R\ddot{R}}{\dot{R}^2} = -1 + \frac{K}{1 + R^k}, \quad (15)$$

where R is the average scale factor, $k > 0$ is a constant which yields a solution

$$R = (\alpha_1 e^{\alpha_2 kt} - 1)^{1/k}, \quad (16)$$

where α_1 and α_2 are constants of integration.

For the metric (1), the scale factor R is given by

$$R = (AB^2)^{1/3}. \quad (17)$$

With the help of equations (16) and (17), we obtain

$$A = (\alpha_1 e^{\alpha_2 kt} - 1)^{\frac{3m}{k(m+2)}}, \quad (18)$$

$$B = (\alpha_1 e^{\alpha_2 kt} - 1)^{\frac{3}{k(m+2)}}. \quad (19)$$

Using equations (18) and (19), the metric (1) takes the form

$$ds^2 = dt^2 - (\alpha_1 e^{\alpha_2 kt} - 1)^{\frac{6m}{k(m+2)}} dr^2 - (\alpha_1 e^{\alpha_2 kt} - 1)^{\frac{6}{k(m+2)}} [d\theta^2 + \sin^2 \theta d\phi^2] \quad (20)$$

Equation (20) represents Kantowski-Sachs minimally interacting holographic dark energy cosmological model with special form of deceleration parameter in Saez-Ballester theory of gravitation.

4. PHYSICAL PROPERTIES OF THE MODEL:

The physical quantities such as spatial volume V , Hubble parameter H , expansion scalar θ , mean anisotropy A_m , shear scalar σ^2 , matter energy density ρ_m , holographic density ρ_λ , holographic pressure p_λ , equation of state parameter ω are obtained as follows:

The spatial volume is in the form

$$V = (\alpha_1 e^{\alpha_2 kt} - 1)^{3/k}. \quad (21)$$

The Hubble parameter is given by

$$H = \alpha_1 \alpha_2 e^{\alpha_2 kt} (\alpha_1 e^{\alpha_2 kt} - 1)^{-1}. \quad (22)$$

The expansion scalar is

$$\theta = 3H = 3\alpha_1 \alpha_2 e^{\alpha_2 kt} (\alpha_1 e^{\alpha_2 kt} - 1)^{-1}. \quad (23)$$

The mean anisotropy parameter is

$$A_m = \frac{2(m-1)^2}{(m+2)^2} = \text{constan } t \quad (\neq 0, \text{ for } m \neq 1). \quad (24)$$

The shear scalar is

$$\sigma^2 = \frac{3\alpha_1^2 \alpha_2^2 (m-1)^2}{(m+2)^2} (\alpha_1 e^{-\alpha_2 kt} - 1)^{-2}. \quad (25)$$

It is observed that

$$\lim_{t \rightarrow \infty} \frac{\sigma^2}{\theta^2} = \frac{(m-1)^2}{3(m+2)^2}, \quad (\neq 0, \text{ for } m \neq 1). \quad (26)$$

The gauge function is given by

$$\phi = \phi_0 c^{\alpha/3} (\alpha_1 e^{\alpha_2 kt} - 1)^{-\alpha/k} \quad (27)$$

The matter energy density is given by

$$\rho_m = c_1 (\alpha_1 e^{\alpha_2 kt} - 1)^{-3/k} \quad (28)$$

The holographic energy density is given by

$$\rho_\lambda = \frac{9\alpha_1^2 \alpha_2^2}{(m+2)^2} (2m+1) e^{2\alpha_2 kt} (\alpha_1 e^{-\alpha_2 kt} - 1)^{-2} + (\alpha_1 e^{-\alpha_2 kt} - 1)^{\frac{-6}{k(m+2)}} + \frac{\omega}{2} \phi_0^{n+2} \alpha^2 \alpha_1^2 \alpha_2^2 e^{2\alpha_2 kt} (\alpha_1 e^{\alpha_2 kt} - 1)^{\frac{\alpha(n-2)-2k}{k}} - c_1 (\alpha_1 e^{\alpha_2 kt} - 1)^{-3/k} \quad (29)$$

The holographic pressure is given by

$$p_\lambda = \left(2k - \frac{9}{m+2}\right) \frac{3\alpha_1^2 \alpha_2^2}{(m+2)} e^{2\alpha_2 kt} (\alpha_1 e^{\alpha_2 kt} - 1)^{-2} - \alpha_1 (\alpha_1 e^{\alpha_2 kt} - 1)^{\frac{-6}{k(m+2)}} - \frac{6\alpha_1 \alpha_2^3}{(m+2)} e^{\alpha_2 kt} (\alpha_1 e^{\alpha_2 kt} - 1)^{-1} + \frac{\omega}{2} \phi_0^{n+2} \alpha^2 \alpha_1^2 \alpha_2^2 e^{2\alpha_2 kt} (\alpha_1 e^{\alpha_2 kt} - 1)^{\frac{\alpha(n-2)-2k}{k}} \quad (30)$$

The Eos parameter ω is given by

$$\omega = \frac{\rho_\lambda}{p_\lambda} = \frac{\frac{9\alpha_1^2 \alpha_2^2}{(m+2)^2} (2m+1) e^{2\alpha_2 kt} (\alpha_1 e^{-\alpha_2 kt} - 1)^{-2} + (\alpha_1 e^{-\alpha_2 kt} - 1)^{\frac{-6}{k(m+2)}} + \frac{\omega}{2} \phi_0^{n+2} \alpha^2 \alpha_1^2 \alpha_2^2 e^{2\alpha_2 kt} (\alpha_1 e^{\alpha_2 kt} - 1)^{\frac{\alpha(n-2)-2k}{k}} - c_1 (\alpha_1 e^{\alpha_2 kt} - 1)^{-3/k}}{\left(2k - \frac{9}{m+2}\right) \frac{3\alpha_1^2 \alpha_2^2}{(m+2)} e^{2\alpha_2 kt} (\alpha_1 e^{\alpha_2 kt} - 1)^{-2} - \alpha_1 (\alpha_1 e^{\alpha_2 kt} - 1)^{\frac{-6}{k(m+2)}} - \frac{6\alpha_1 \alpha_2^3}{(m+2)} e^{\alpha_2 kt} (\alpha_1 e^{\alpha_2 kt} - 1)^{-1} + \frac{\omega}{2} \phi_0^{n+2} \alpha^2 \alpha_1^2 \alpha_2^2 e^{2\alpha_2 kt} (\alpha_1 e^{\alpha_2 kt} - 1)^{\frac{\alpha(n-2)-2k}{k}}} \quad (31)$$

The matter density parameter Ω_m and holographic dark energy density parameter Ω_λ are given by

$$\Omega_m = \frac{\rho_m}{3H^2} = \frac{c_1}{3\alpha_1^2 \alpha_2^2} e^{-2\alpha_2 kt} (\alpha_1 e^{\alpha_2 kt} - 1)^{\frac{2k-3}{k}} \quad (32)$$

$$\Omega_\lambda = \frac{\rho_\lambda}{3H^2} = \frac{\frac{9\alpha_1^2 \alpha_2^2}{(m+2)^2} (2m+1) e^{2\alpha_2 kt} (\alpha_1 e^{-\alpha_2 kt} - 1)^{-2} + (\alpha_1 e^{-\alpha_2 kt} - 1)^{\frac{-6}{k(m+2)}} + \frac{\omega}{2} \phi_0^{n+2} \alpha^2 \alpha_1^2 \alpha_2^2 e^{2\alpha_2 kt} (\alpha_1 e^{\alpha_2 kt} - 1)^{\frac{\alpha(n-2)-2k}{k}} - c_1 (\alpha_1 e^{\alpha_2 kt} - 1)^{-3/k}}{3\alpha_1^2 \alpha_2^2 (\alpha_1 e^{\alpha_2 kt} - 1)^{-2}} \quad (33)$$

The overall density parameter as

$$\Omega = \Omega_m + \Omega_\lambda = \frac{c_1}{3\alpha_1^2 \alpha_2^2} e^{-2\alpha_2 kt} (\alpha_1 e^{\alpha_2 kt} - 1)^{\frac{2k-3}{k}} + \frac{9\alpha_1^2 \alpha_2^2}{(m+2)^2} (2m+1) e^{2\alpha_2 kt} (\alpha_1 e^{-\alpha_2 kt} - 1)^{-2} + (\alpha_1 e^{-\alpha_2 kt} - 1)^{\frac{-6}{k(m+2)}} + \frac{\omega}{2} \phi_0^{n+2} \alpha^2 \alpha_1^2 \alpha_2^2 e^{2\alpha_2 kt} (\alpha_1 e^{\alpha_2 kt} - 1)^{\frac{\alpha(n-2)-2k}{k}} - c_1 (\alpha_1 e^{\alpha_2 kt} - 1)^{-3/k} \quad (34)$$

From equation (21), the spatial volume is finite i.e. the universe starts evolving with some finite volume at $t = 0$ and expands with cosmic time t . From equations (18) and (19), the spatial scale factors are not zero for any value of t and hence the model does not have singularity. From equation (24), the mean anisotropy parameter A_m is constant throughout the evolution of the universe. Hence the model is anisotropic throughout the evolution of the universe. From equations (28) and (29), the matter energy density (ρ_m) and holographic energy density (ρ_λ) starts with finite density and increases as time increases.

CONCLUSION: Kantowski-Sachs cosmological model has been discussed in the scalar-tensor theory of gravitation proposed by Saez-Ballester when the source for energy momentum tensor is minimally interacting holographic dark energy. The solution of the field equations have obtained by choosing the special form of deceleration parameter $q = -1 + \frac{k}{1+R^2}$. It is observed that in early phase of universe, the value of deceleration parameter is positive while as $t \rightarrow \infty$, the value of $q = -1$. Hence the universe has a decelerated expansion in the past and accelerated expansion at present which is in good agreement with the recent observations of SN Ia. Since the mean anisotropy parameter A_m is constant throughout the evolution of the universe, hence the model is anisotropic throughout the evolution of the universe. The spatial volume increases with increase in time representing that the model is expanding. We hope that our model will be useful in the discussion of structure formation in the early universe and an accelerating expansion of the universe at present.

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